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Mrs Collett
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Name: ……………………………………………

Teacher:…………………………………………

Pymble Ladies' College

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION 2014

Mathematics

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours.
- Use pencil for Questions 1-10.
- Write using black or blue pen for Questions 11-16. Black pen is preferred.
- Board approved calculators may be used.
- A table of standard integrals is provided at the back of this paper.
- In Questions 11-16, show relevant mathematical reasoning and/or calculations.

Total Marks – 100

Section I Pages 1-4

10 marks
- Attempt Questions 1-10
- Allow about 15 mins for this section

Section II Pages 5-15

90 marks
- Attempt Questions 11-16
- Allow about 2 hours 45 minutes for this section

Mark	**/100**
Highest Mark	**/100**
Rank	

Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

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1 What is the period of $y = 5\sin 2x$?

(A) π

(B) 2π

(C) $\frac{\pi}{2}$

(D) 5

2 What is the value of x if $8(x-3)^3 - 1 = 0$?

(A) $3-\frac{1}{2}$

(B) $3+\frac{1}{2}$

(C) $3\pm\frac{1}{2}$

(D) 5

3 What are the values of m that will give the equation $mx^2 + 6x - 3 = 0$ two real and different roots?

(A) $m \leq -3$

(B) $m \geq -3$

(C) $m > -3$

(D) $m < -3$

Multiple Choice (continued).

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4 What is the gradient of the normal to the curve $y = 2x^2 - 5x + 1$ at the point $(2, -1)$?

(A) $\frac{1}{3}$

(B) $-\frac{1}{3}$

(C) 3

(D) -3

5 What is the derivative of $f(x) = \ln(\cos x)$?

(A) $f'(x) = -\tan x$

(B) $f'(x) = \tan x$

(C) $f'(x) = \frac{1}{\cos x}$

(D) $f'(x) = -\frac{1}{\sin x}$

6 What are the solutions of $2\sin x + \sqrt{3} = 0$ in the domain $0 \le x \le 2\pi$?

(A) $\frac{\pi}{3}, \frac{2\pi}{3}$

(B) $\frac{2\pi}{3}, \frac{5\pi}{3}$

(C) $\frac{\pi}{3}, \frac{4\pi}{3}$

(D) $\frac{4\pi}{3}, \frac{5\pi}{3}$

7 Given $\log_{10} y = 2 - \log_{10} x$, which expression is equivalent to y?

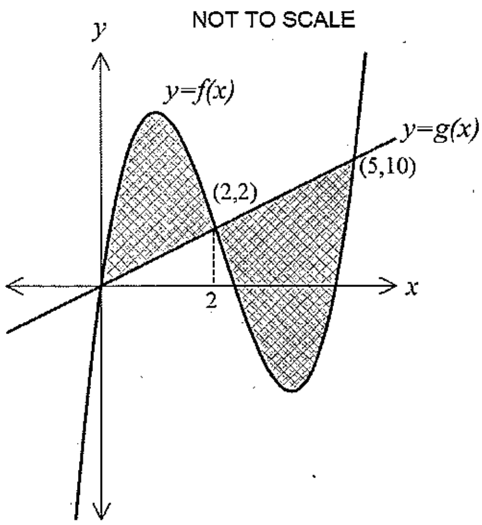
(A) $y = \log_{10}(2) - x$

(B) $y = 2 - x$

(C) $y = \dfrac{100}{x}$

(D) $y = 100 - x$

8 Which expression using integral notation is equivalent to the area of the shaded regions?



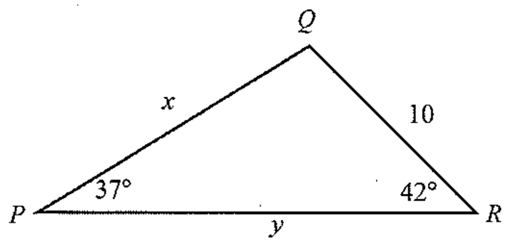
(A) $\int_0^5 \{f(x) - g(x)\}dx$

(B) $\int_0^2 \{g(x) - f(x)\}dx + \int_2^5 \{f(x) - g(x)\}dx$

(C) $\int_0^2 \{f(x) - g(x)\}dx + \int_2^5 \{f(x) - g(x)\}dx$

(D) $\int_0^2 \{f(x) - g(x)\}dx + \int_2^5 \{g(x) - f(x)\}dx$

9 PQR is a triangle with side lengths x, 10 and y, as shown below.
In this triangle, angle $RPQ = 37°$ and angle $QRP = 42°$.



Which one of the following expressions is correct for triangle PQR?

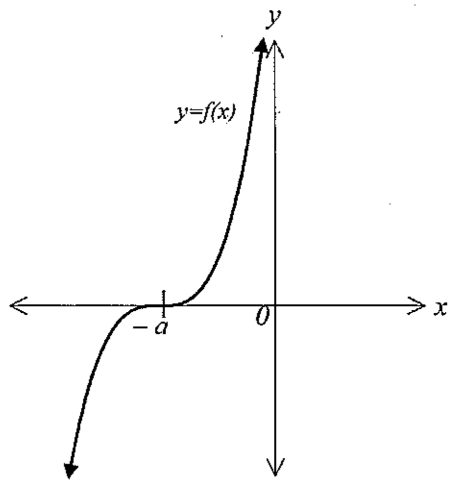
(A) $x = \dfrac{10}{\sin 37°}$

(B) $x = 10 \times \dfrac{\sin 42°}{\sin 37°}$

(C) $y = 10 \times \dfrac{\sin 37°}{\sin 101°}$

(D) $10^2 = x^2 + y^2 - 2xy\cos 42°$

10 At $x = -a$, which of the following correctly describes the graph of $y = f(x)$?



(A) $f(-a) = 0,\ f'(-a) > 0$

(B) $f'(-a) = 0,\ f''(-a) = 0$

(C) $f(0) = -a,\ f'(-a) > 0$

(D) $f'(-a) > 0,\ f''(-a) = 0$

Section II

90 marks
Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section.

Answer each question in the appropriate writing booklet. Extra booklets are available.

In Questions 11 – 16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Solve the equation $|3+5x|=2$. **2**

(b) Show that $3\sqrt{5}-2\sqrt{2}$ is a square root of $53-12\sqrt{10}$. **2**

(c) Differentiate $(5-\cos 2x)^4$. **2**

(d) Find a primitive of $\sec^2 x-3$. **2**

Question 11 continues on page 6.

(e) α and β are the roots of $x^2 - 4x + 1 = 0$.

(i) Find $\alpha\beta$. **1**

(ii) Hence, prove $\alpha + \dfrac{1}{\alpha} = 4$. **1**

(f) Find the coordinates of the focus of the parabola $x^2 = -32(y-2)$. **2**

(g) A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are $3°$ and $5°$. Find the value of n. **3**

End of Question 11

Question 12 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Given that $y = \dfrac{x^2}{\tan 4x}$, find $\dfrac{dy}{dx}$. **2**

(b) Find $\int \sqrt{7x-2}\, dx$. **2**

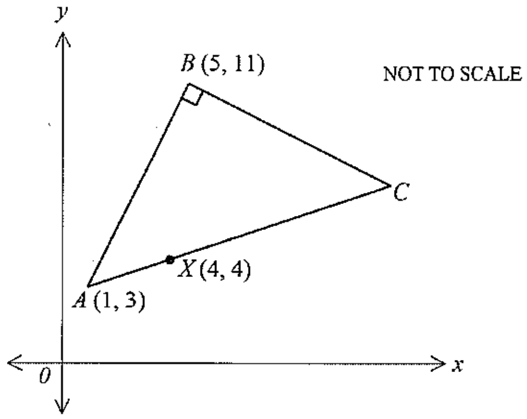
(c) In a geometric series, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

(i) the first term, **2**

(ii) the sum to infinity of the series. **1**

Question 12 continues on page 8.

(d)



The diagram above shows a triangle ABC in which A has coordinates $(1, 3)$, B has coordinates $(5, 11)$ and angle ABC is $90°$. The point $X\ (4, 4)$ lies on AC. Find

(i) the gradient of AB. **1**

(ii) the equation of BC. **2**

(iii) the coordinates of C. **3**

(e) It is given that $f(x) = \dfrac{1}{x^3} - x^3$. Show that $f(x)$ is a decreasing function. **2**

End of Question 12

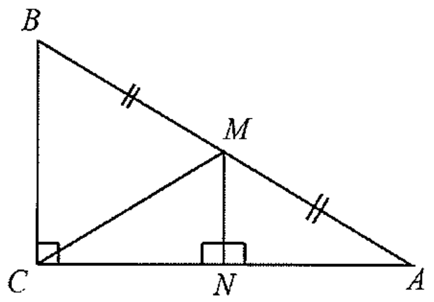
Question 13 (15 marks) Use a **separate** writing booklet. **Marks**

(a) A function is given by $f(x) = x^3 - 3x^2 - 9x + 11$.

(i) Find the coordinates of the stationary points of $f(x)$ and determine their nature. **3**

(ii) Hence, sketch the graph $y = f(x)$ showing all stationary points and the y - intercept. **2**

(b) In the diagram, M is the midpoint of AB. $\angle ACB = \angle MNA = 90°$.
Copy the diagram into your booklet.



Prove that

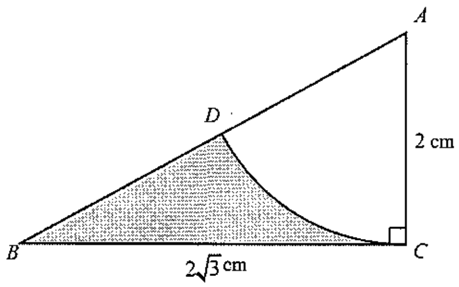
(i) $MN = \frac{1}{2}BC$. **2**

(ii) ΔAMN and ΔCMN are congruent. **2**

(iii) $CM = \frac{1}{2}AB$ **1**

Question 13 continues on page 10.

(c)



In the diagram, D lies on the side AB of the triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC. Find the area of the shaded region BDC, giving your answer in terms of π and $\sqrt{3}$. **2**

(d) A curve is such that $\dfrac{d^2y}{dx^2} = 4e^{-2x}$. Given that $\dfrac{dy}{dx} = 3$ when $x = 0$ and that the curve passes through the point $\left(2,\ e^{-4}\right)$, find the equation of the curve. **3**

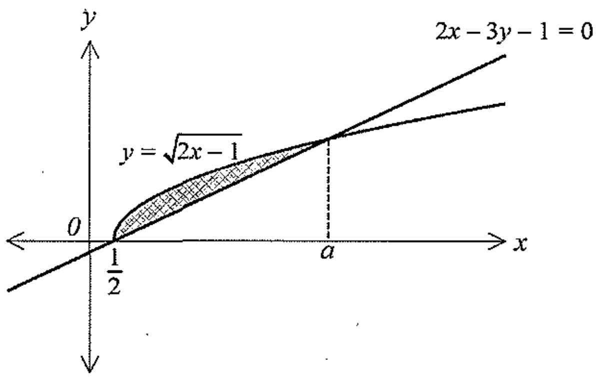
End of Question 13

Question 14 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Prove that $\dfrac{\sin A}{1+\cos A}+\dfrac{1+\cos A}{\sin A}=2\text{cosec}A$ **3**

(b) The curves $y=\sqrt{2x-1}$ and $2x-3y-1=0$ are drawn below.

They intersect at $x=\dfrac{1}{2}$ and $x=a$ as indicated on the diagram.



(i) Show that $a=5$. **2**

(ii) Find, showing all necessary working, the area of the shaded region. **3**

(c) The temperature $T°C$ of an object in a room, after t minutes, satisfies the differential equation

$$\frac{dT}{dt}=k(T-22), \text{ where } k \text{ is a constant.}$$

(i) Show that $T=Ae^{kt}+22$, satisfies the differential equation. **1**

(ii) When $t=0, T=100,$ and when $t=15, T=70$.

(α) Use this information to find the values of A and k. **3**

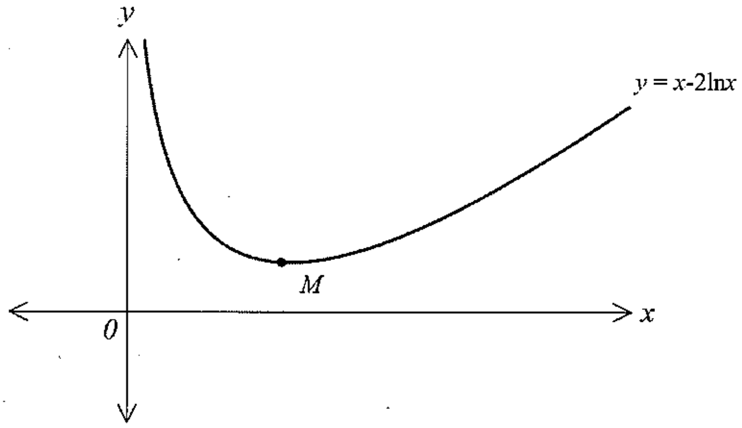
(β) Hence find the value of t when $T=40$, correct to 1 decimal place. **3**

End of Question 14

Question 15 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Evaluate $\int_2^3 \frac{x^2}{x^3-2}\,dx.$ **2**

(b)



The above diagram shows the curve $y = x - 2\ln x$ and its minimum point M.

(i) Find the x coordinate of M. **2**

(ii) Use 2 applications of the trapezoidal rule to estimate the value of **3**

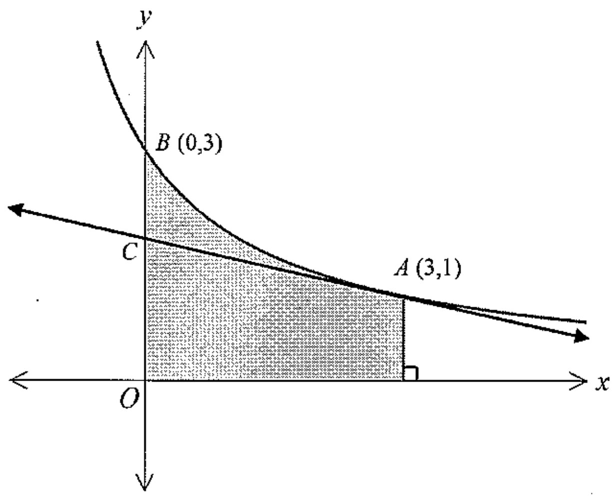
$$\int_2^4 (x - 2\ln x)\,dx.$$

Give your answer to correct to 2 decimal places.

(iii) State, with a reason, whether the trapezoidal rule gives an under-estimate or an over-estimate of the true value of the integral in part (ii). **1**

Question 15 continues on page 13.

(c)



The above diagram shows part of the curve $y = \dfrac{9}{2x+3}$, crossing the y-axis at the point B (0,3).

The point A on the curve has coordinates (3,1).

The tangent to the curve at A crosses the y-axis at C.

(i) Find the equation of the tangent to the curve at A. **3**

(ii) Determine, showing all necessary working, whether C is nearer to the point B or to the point O. **1**

(iii) Find the exact volume obtained when the shaded region is rotated through $360°$ about the x-axis. Show all necessary working. **3**

End of Question 15

Question 16 (15 marks) Use a **separate** writing booklet. **Marks**

(a) Solve $4e^{2x} - e^{x} = 0$. **2**

(b) A particle moves in a straight line and at time t it has velocity v, where

$$v = 3t^2 - 2\sin 3t + 6$$

(i) Find an expression for the acceleration of the particle at time t. **1**

(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. **1**

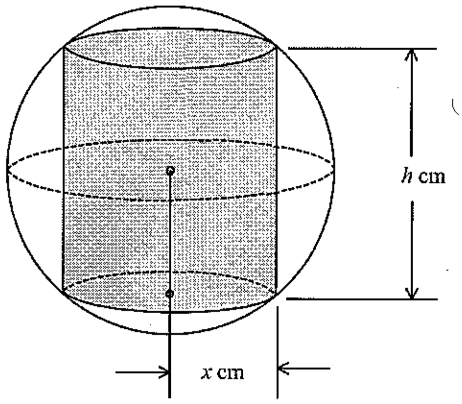
(iii) When $t = 0$, the particle is at the origin.
Find an expression for the displacement of the particle from the origin at time t. **2**

(c) (i) Find $\frac{d}{dx}\left(e^{\cos 2x}\right)$. **1**

(ii) Hence, find $\int x + \sin 2x \, e^{\cos 2x} dx$. **2**

Question 16 continues on page 15.

(d) A machinist has a spherical ball of brass with diameter 10 cm. The ball is placed in a lathe and machined into a cylinder.



(i) If the cylinder has radius x cm, show that the cylinder's volume is given by **2**

$$V(x) = \pi x^2\sqrt{100 - 4x^2}\ \text{cm}^3.$$

(ii) Hence, find the dimensions of the cylinder of largest volume which can be made. **4**

End of paper

Mathematics Trial HSC 2014

Solutions

1. A
2. B
3. C
4. B
5. A
6. D
7. C
8. D
9. B
10. B

Question 11

a) $|3+5x| = 2$

$3+5x = 2$ or $-(3+5x) = 2$

$5x = -1$ $\qquad$ $3+5x = -2$

$x = -\frac{1}{5}$ or $5x = -5$

(1 mark) $\qquad$ $x = -1$ (1 mark)

$\therefore x = -1$ or $-\frac{1}{5}$

b) $(3\sqrt{5} - 2\sqrt{2})^2 = 45 - 2(2\sqrt{2})(3\sqrt{5}) + 8$ (1 mark)

$= 53 - 12\sqrt{10}$

$\therefore \sqrt{53-12\sqrt{10}} = \pm(3\sqrt{5} - 2\sqrt{2})$

$\therefore$ $3\sqrt{5} - 2\sqrt{2}$ is a square root of $53-12\sqrt{10}$

(1 mark)

c) $\frac{d}{dx}(5-\cos 2x)^4$

$= 4(5-\cos 2x)^3 \cdot 2\sin 2x$ (1 mark)

$= 8\sin 2x\,(5-\cos 2x)^3$ (1 mark)

d) $\int(\sec^2 x - 3)\,dx$

$= \tan x - 3x + C$

(1 mark) $\qquad$ (1 mark)

e) $x^2 - 4x + 1 = 0$

i) $\alpha\beta = \frac{c}{a}$

$\alpha\beta = 1$ (1 mark)

ii) From (i) $\beta = \frac{1}{\alpha}$ and $\alpha + \beta = 4$

Sub $\beta = \frac{1}{\alpha}$ into $\alpha + \beta = 4$

$\therefore \alpha + \frac{1}{\alpha} = 4$ (1 mark)

Question 11 (cont'd)

(f) $x^2 = -32(y-2)$

$x^2 = -4a(y-2)$

$a = 8$ (1 mark)

Vertex $(0, 2)$

$\therefore$ Focus $(0, -6)$ (1 mark)

g) $3° + 5° + 7° + \ldots = 360°$ (AP)

$a = 3 \quad d = 2 \quad S_n = 360°$ (1 mark)

$S_n = \frac{n}{2}(2a + (n-1)d)$

$360 = \frac{n}{2}(6 + (n-1)2)$

$720 = n(6 + 2n - 2)$

$720 = n(4 + 2n)$

$\therefore 2n^2 + 4n - 720 = 0$ (1 mark)

$n^2 + 2n - 360 = 0$

$(n+20)(n-18) = 0$

$n = 18$ or -20

but $n > 0$

$\therefore n = 18$ (1 mark)

Question 12

a) $y = \dfrac{x^2}{\tan 4x}$

$u = x^2$

$\dfrac{du}{dx} = 2x$

$v = \tan 4x$

$\dfrac{dv}{dx} = 4\sec^2 4x$

$\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\therefore \dfrac{dy}{dx} = \underbrace{\dfrac{\tan 4x \cdot 2x - \overbrace{x^2 \cdot 4\sec^2 4x}^{\text{(1 mark)}}}{\tan^2 4x}}_{\text{(1 mark)}}$

or $\dfrac{dy}{dx} = \dfrac{2x\tan 4x - 4x^2\sec^2 4x}{\tan^2 4x}$

b) $\int \sqrt{7x-2}\, dx$

$= \int (7x-2)^{\frac{1}{2}}\, dx$

$= \dfrac{(7x-2)^{\frac{3}{2}}}{\frac{3}{2}\times 7} + c$

$= \dfrac{2\sqrt{(7x-2)^3}}{21} + c$ (1 mark)

(1 mark)

c) i) $T_2 = 24$ GP $\therefore T_n = ar^{n-1}$

$T_4 = 13\frac{1}{2}$

$ar = 24$ ①

$ar^3 = \dfrac{27}{2}$ ②

② ÷ ① $r^2 = \dfrac{27}{48}$

$r = \pm\sqrt{\dfrac{27}{48}}$

$r = \pm \dfrac{3}{4}$ (1 mark)

since $r > 0$ then $r = \dfrac{3}{4}$ and $a = \dfrac{24}{\frac{3}{4}} = 32$ (1) ④

Q12 (cont'd)

c (i) $S_\infty = \frac{a}{1-r}$

NB $|r|<1$

$= \frac{32}{1-\frac{3}{4}}$

$= \frac{32}{\frac{1}{4}}$

$= 128$ (1 mark)

d i) $M_{AB} = \frac{11-3}{5-1}$

$= 2$

ii) $M_{BC} = -\frac{1}{2}$

$\therefore$ BC is $y - 11 = -\frac{1}{2}(x-5)$ (1 mark)

$2y - 22 = -x + 5$

$x + 2y - 27 = 0$ (1)

(1 mark)

iii) $M_{AC} = \frac{4-3}{4-1}$

$= \frac{1}{3}$

$\therefore$ AC is $y - 3 = \frac{1}{3}(x-1)$

$3y - 9 = x - 1$

$x - 3y + 8 = 0$ (2)

(1) − (2) $5y - 35 = 0$

$y = 7$ (1 mark)

Sub in (1)

$x = 27 - 2(7)$

$x = 13$

$\therefore$ C is $(13, 7)$ (1 mark)

(5)

Question 12 (cont'd)

e) $f(x) = \frac{1}{x^3} - x^3 \quad , x \neq 0$

$= x^{-3} - x^3$

For a decreasing function $f'(x) < 0$

$f'(x) = -3x^{-4} - 3x^2$

$= -\frac{3}{x^4} - 3x^2$

For all x, x^2 and $x^4 > 0$

$\therefore -\frac{3}{x^4} < 0$ and $-3x^2 < 0$.

$\therefore f'(x) < 0$ for all x

$\therefore$ decreasing function.

(1 mark for showing correct $f'(x) < 0$ for decreasing function)

(1 mark for qualifying why function will be < 0 for all x.)

Question 13

a i) $f(x) = x^3 - 3x^2 - 9x + 11$

$f'(x) = 3x^2 - 6x - 9$

For stationary points, $f'(x) = 0$

$\therefore \quad 3x^2 - 6x - 9 = 0$

$3(x^2 - 2x - 3) = 0$

$3(x-3)(x+1) = 0$

$\therefore \quad x = 3$ or -1

$(3, -16) \quad (-1, 16)$ (1 mark)

$f''(x) = 6x - 6$

when $x = -1$ $\quad f''(-1) = -6 - 6$

$= -12$ which is < 0

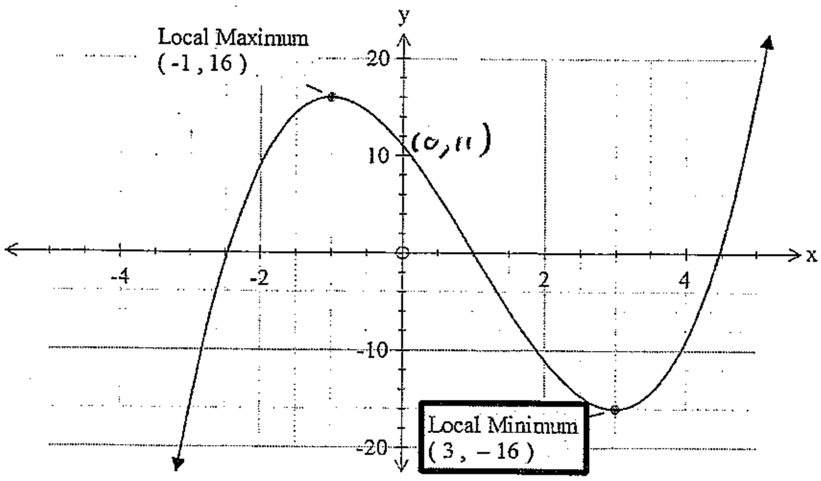
$\therefore$ Maximum turning point at $(-1, 16)$ (1 mark)

when $x = 3$ $\quad f''(3) = 18 - 6$

$= 12$ which is > 0

$\therefore$ Minimum turning point at $(3, -16)$

(1 mark)



(1 mark for points and scale)

(1 mark for shape)

Question 13 (cont'd)

b i) In $\triangle BCA$ and $\triangle MNA$

$\angle BCA = \angle MNA = 90°$ (given)

$\angle A$ is common (1 mark)

$\therefore \triangle BCA \ ||| \ \triangle MNA$ (equiangular)

$\therefore \frac{BA}{MA} = \frac{BA}{\frac{1}{2}BA} = 2$ since M is the midpoint of AB

$\therefore$ The reduction factor is $\frac{1}{2}$ (1 mark)

$\therefore MN = \frac{1}{2}BC$ (ratio of corresponding sides in similar triangles)

ii) $\frac{AM}{AB} = \frac{1}{2}$

$\therefore \frac{AN}{AC} = \frac{1}{2}$ (ratios of corresponding sides in similar triangles)

$AN = \frac{1}{2}AC$

$\therefore AN = CN$ (1 mark)

In $\triangle AMN$ and $\triangle CMN$

$AN = CN$ (shown above)

$MN = MN$ common side (1 mark)

$\angle ANM = \angle CNM = 90°$ (given)

$\therefore \triangle AMN \equiv \triangle CMN$ (SAS)

iii) $CM = MA$ (corresponding sides in congruent triangles from (ii))

$BM = MA$ given

$\therefore CM = BM$ (1 mark)

$CM = \frac{1}{2}BA$ (Given M is the midpoint of AB)

Question 13 (b) (OR)

b) i) $AB = AM + BM$ (M is the midpoint of AB)

$= AM + AM$ $(AM = BM)$

$= 2AM$ —— (1)

In $\triangle AMN$ and $\triangle ABC$,

$\sin A = \frac{MN}{AM} = \frac{BC}{AB}$

$\Rightarrow MN \cdot AB = BC \cdot AM$

$MN \cdot 2AM = BC \cdot AM$

$2MN = BC$

$MN = \frac{1}{2} BC$ (1 mark)

(2)

ii) $\angle ANM = \angle ACB = 90^\circ$ (given)

Since corresponding angles are equal, $MN \parallel BC$.

$\frac{AM}{BM} = \frac{AN}{CN}$ (interval parallel to a side of a triangle divides other sides in same ratio)

$\frac{AN}{CN} = \frac{AM}{AM}$ $(AM = BM)$

$= 1$

$AN = CN$ —— (2) (1 mark)

In $\triangle AMN$ and $\triangle CMN$;

$MN = MN$ (common)

$\angle ANM = \angle CNM = 90^\circ$ (given)

$AN = CN$ (from (2))

$\therefore \triangle AMN \equiv \triangle CMN$ (SAS) (1 mark)

iii) $CM = AM$ (corresponding sides of congruent triangles)

$AB = 2AM$ (from (1))

$\Rightarrow AM = \frac{1}{2} AB$

$CM = \frac{1}{2} AB$ (1 mark)

(8b)

Question 13 (cont'd)

c) $\tan A = \frac{2\sqrt{3}}{2}$

$= \sqrt{3}$

$\therefore A = \frac{\pi}{3}$

Area BDC = Area $\triangle ABC$ − Area sector ADC

$= \frac{1}{2} \times 2\sqrt{3} \times 2 - \frac{1}{2} \times 2^2 \times \frac{\pi}{3}$ (1 mark)

$= 2\sqrt{3} - \frac{2\pi}{3}$ cm^2 (1 mark)

d) $\frac{d^2y}{dx^2} = 4e^{-2x}$

$\frac{dy}{dx} = \frac{-4}{2} e^{-2x} + c$ (1 mark)

When $x = 0$ $\frac{dy}{dx} = 3$

$\therefore 3 = -2e^0 + c$

$c = 5$

$\therefore \frac{dy}{dx} = -2e^{-2x} + 5$

$y = e^{-2x} + 5x + k$ (1 mark)

When $x = 2$ $y = e^{-4}$

$\therefore e^{-4} = e^{-4} + 5 \times 2 + k$

$\therefore k = -10$

$\therefore y = e^{-2x} + 5x - 10$ (1 mark)

Question 14

a) $LHS = \frac{\sin A}{1+\cos A} + \frac{1+\cos A}{\sin A}$

$= \frac{\sin^2 A + (1+\cos A)^2}{(1+\cos A)\sin A}$

$= \frac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{(1+\cos A)\sin A}$ (1 mark)

$= \frac{2 + 2\cos A}{(1+\cos A)\sin A}$ since $\sin^2 A + \cos^2 A = 1$

$= \frac{2(1+\cos A)}{\sin A(1+\cos A)}$ (1 mark)

$= \frac{2}{\sin A}$

$= 2\operatorname{cosec} A$ (1 mark)

$= RHS.$

$\therefore LHS = RHS.$

b) $x = 5$ sub in $y = \sqrt{2x-1}$

$y = \sqrt{2\times5-1}$

$y = 3$ (1 mark) $x > 0$

OR

$x = 5$ Sub in $2x - 3y - 1 = 0$ (1 for simultaneous equations)

$2\times5 - 3y - 1 = 0$

$3y = 10 - 1$ (1 Correct follow

$y = 3$ (1 mark) through to show $x=5$ as one of 2 solutions)

Both equations equal 3 when $x = 5$

$\therefore$ (2nd pt of intersection is $(5, 3)$

(10)

Question 14 cont'd

b ii) $A = \int_{\frac{1}{2}}^{5} (2x-1)^{\frac{1}{2}}\,dx - \frac{1}{2} \times \frac{9}{2} \times 3$

$= \left[\frac{2(2x-1)^{\frac{3}{2}}}{3 \times 2} \right]_{\frac{1}{2}}^{5} - \frac{27}{4}$

$= \frac{9^{\frac{3}{2}}}{3} - 0 - \frac{27}{4}$ (1 mark correct integration of $(2x-1)^{\frac{1}{2}}$)

$= 9 - \frac{27}{4}$ (1 mark correct sub of both expressions)

$= \frac{9}{4}$ sq units. (1 mark evaluation of substituted value

c) i) If $T = Ae^{kt} + 22$ or $Ae^{kt} = T - 22$

$\frac{dT}{dt} = kAe^{kt}$

but $Ae^{kt} = T - 22$ (1 mark)

$\therefore \frac{dT}{dt} = k(T-22)$ as required.

ii) α) $100 = Ae^{0k} + 22$

$A = 100 - 22$

$A = 78$ (1 mark)

$\therefore 70 = Ae^{15k} + 22$

$70 - 22 = 78e^{15k}$

$48 = 78e^{15k}$

$e^{15k} = \frac{48}{78}$

$e^{15k} = \frac{8}{13}$ (1 mark)

$15k = \ln \frac{8}{13}$

$\therefore k = \frac{1}{15} \ln \frac{8}{13}$ (1 mark)

Question 14 (Cont'd)

B) When $T = 40$ $\quad 40 = 78e^{kt} + 22$

$$e^{kt} = \frac{40-22}{78}$$

$$e^{kt} = \frac{3}{13} \qquad \text{(1 mark)}$$

$$\therefore kt = \ln \frac{3}{13}$$

$$t = \frac{1}{k} \ln \frac{3}{13}$$

$$= \frac{15 \ln \frac{3}{13}}{\ln \frac{8}{13}} \qquad \text{(1 mark)}$$

$$= 45.3 \qquad \text{(1 mark)}$$

Question 15

a) $\int_2^3 \frac{x^2}{x^3-2}\,dx$

$= \left[\frac{1}{3}\ln(x^3-2)\right]_2^3$ (1 mark)

$= \frac{1}{3}\left[\ln 25 - \ln 6\right]$ (1 mark)

$= \frac{1}{3}\left[2\ln 5 - \ln 6\right]$

b) i) $y = x - 2\ln x$

$\frac{dy}{dx} = 1 - \frac{2}{x}$ (1 mark)

Stationary points occur at $y' = 0$

$1 - \frac{2}{x} = 0$

$\frac{2}{x} = 1$

$x = 2$ (1 mark)

ii)

x	2	3	4
$f(x)$	$2-2\ln 2$	$3-2\ln 3$	$4-2\ln 4$

(1 mark)

(1 mark)

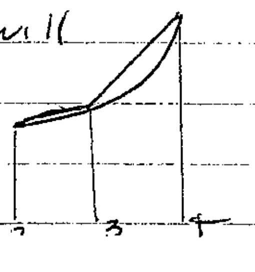
$\int_2^4 (x-2\ln x)\,dx \doteq \frac{1}{2}\left[2-2\ln 2+2(3-2\ln 3)+4-2\ln 4\right]$

$= 1.723\ldots$ (1 mark)

$= 1.72$ to 2 dp.

iii) Since the curve is concave up for $2 \le x \le 4$, the trapezoidal rule will give an over estimate.

(1 mark)



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Question 15 (cont'd)

c) $y = \frac{9}{2x+3}$

$y = 9(2x+3)^{-1}$

$\frac{dy}{dx} = -9 \times 2(2x+3)^{-2}$ (1 mark)

$= \frac{-18}{(2x+3)^2}$

At A when $x = 3$

$\frac{dy}{dx} = \frac{-18}{9^2}$

$= \frac{-2}{9}$ (1 mark)

Tangent at A is: $y - 1 = -\frac{2}{9}(x-3)$

$9y - 9 = -2x + 6$

$2x + 9y - 15 = 0$ (1 mark)

ii) AC cuts y axis when $x = 0$

$2(0) + 9y - 15 = 0$

$9y = 15$

$y = \frac{15}{9}$

$y = \frac{5}{3}$

Distance from ~~OC~~ O to C is $\frac{5}{3} = 1\frac{2}{3}$

Distance from C to B is $3 - \frac{5}{3} = 1\frac{1}{3}$

$\therefore$ C is closer to B. (1 mark)

Question 15 (cont'd)

c iii) $V = \pi \int_0^3 y^2 \, dx$

$= \pi \int_0^3 \left[\frac{9}{(2x+3)} \right]^2 dx$ (1 mark)

$= 81\pi \int_0^3 (2x+3)^{-2} \, dx$

$= 81\pi \left[\frac{-1}{2(2x+3)} \right]_0^3$ (1 mark)

$= \frac{-81\pi}{2} \left(\frac{1}{6+3} - \frac{1}{3} \right)$

$= \frac{-81\pi}{2} \left(\frac{1}{9} - \frac{1}{3} \right)$

$= 9\pi$ cubic units. (1 mark)

Question 16

a) $4e^{2x} - e^{x} = 0$

$\therefore 4(e^{x})^2 - e^{x} = 0$

Let $m = e^{x}$

$4m^2 - m = 0$

$m(4m-1) = 0$

$\therefore 4m - 1 = 0$ or $m = 0$

$m = \frac{1}{4}$ no sol^n

$\therefore e^{x} = \frac{1}{4}$ (as $e^{x} > 0$)

(1 mark)

$\ln e^{x} = \ln \frac{1}{4}$

$x = \ln \frac{1}{4}$ (1 mark)

$x = \ln 2^{-2}$

$x = -2\ln 2$

b) i) $v = 3t^2 - 2\sin 3t + 6$

$a = 6t - 6\cos 3t$ (1 mark)

ii) When $t = \frac{\pi}{3}$ $a = 6\left(\frac{\pi}{3}\right) - 6\cos\left(3 \times \frac{\pi}{3}\right)$

$= 2\pi - 6\cos\pi$

$= 2\pi - 6(-1)$ (1) mark

$= 2\pi + 6$

iii) Displacement $x = t^3 + \frac{2}{3}\cos 3t + 6t$ (1 mark)

When $t=0$ $x=0$

$0 = 0 + \frac{2}{3} \times 1 + 0 + c$

$c = -\frac{2}{3}$

$\therefore x = t^3 + \frac{2}{3}\cos 3t + 6t - \frac{2}{3}$ (1 mark)

Question 16 (cont'd)

c) i) $\frac{d}{dx} e^{\cos 2x} = -2\sin 2x \, e^{\cos 2x}$ (1 mark)

ii) $\int x + \sin 2x \, e^{\cos 2x} \, dx$

$= \frac{x^2}{2} - \frac{1}{2}\int -2\sin 2x \, e^{\cos 2x} \, dx$

$= \frac{x^2}{2} - \frac{1}{2} e^{\cos 2x} + C$ (2 marks)

(1 mark for partial sol^n)

d) i) $V = A \times H$

$= \pi r^2 \times H$

$= \pi \times x^2 \times h$

Now $x^2 + \left(\frac{h}{2}\right)^2 = 25$ (1 mark)

$\therefore \frac{h^2}{4} = 25 - x^2$

$h^2 = 100 - 4x^2$

$h = \sqrt{100 - 4x^2}$ $h > 0$ (1 mark)

$\therefore V(x) = \pi x^2 \sqrt{100 - 4x^2}$ cm^3.

ii)

$V'(x) = 2\pi x \sqrt{100 - 4x^2} + \pi x^2 . \frac{1}{2}(100 - 4x^2)^{-\frac{1}{2}} . -8x$

$= 2\pi x \sqrt{100 - 4x^2} - \frac{4\pi x^3}{\sqrt{100 - 4x^2}}$ (1 mark)

$= \frac{2\pi x(100 - 4x^2) - 4\pi x^3}{\sqrt{100 - 4x^2}}$

For maximum volume $V'(x) = 0$

Question 16 (cont'd)

d)

$\therefore 2\pi x(100-4x^2-2x^2)=0$

$\therefore x=0$ or $100-6x^2=0$

$x^2=\frac{100}{6}$

$\therefore x=\pm\frac{5\sqrt{2}}{\sqrt{3}}$

$x=\pm\frac{5\sqrt{6}}{3}$ (1 mark)

but $x>0$ $\therefore x=\frac{5\sqrt{6}}{3}$

Check if $x=\frac{5\sqrt{6}}{3}$ gives maximum volume

x	0	1	$\frac{5\sqrt{6}}{3}$	4.5
$V'(x)$	0	$\frac{188\pi}{\sqrt{96}}$	0	$\frac{171\pi-364.5\pi}{\sqrt{19}}$
$V'(x)$	0	>0	0	<0

(1 mark)

$\therefore x=\frac{5\sqrt{6}}{3}$ gives a maximum value.

Dimensions Radius $\frac{5\sqrt{6}}{3}$ cm

Height $\sqrt{100-4\left(\frac{5\sqrt{6}}{3}\right)^2}$

$=\sqrt{\frac{100}{3}}$

$=\frac{10}{\sqrt{3}}$ (1 mark)

$\therefore h=\frac{10\sqrt{3}}{3}$ cm